EQUIPARTITION OF ENERGY AT A HORIZON AND THE IMMIRZI PARAMETER OF LOOP QUANTUM GRAVITY

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ABSTRACT: *The quantization of geometry arising from loop quantum gravity (LQG) is undetermined up to a free parameter γ, called the Immirzi parameter. The value of this parameter is often fixed by comparing the LQG result for the black hole entropy with the Bekenstein-Hawking entropy formula and the quasinormal mode spectrum of the black hole. Some authors have suggested that energy is spread over the degrees of freedom at a general holographic sphere according to the equipartition rule. We find that one arrives at a new value if the LQG quantum states at the horizon are subjected to obey the equipartition rule.*

Keywords: Quantum gravity, black hole, loop quantum gravity, Immirzi parameter, the holographic principle

INTRODUCTION

Exploiting the resounding similarity of the laws of thermodynamics to the black hole physics uncovered by Hawking and others [1- 4], and using purely imaginative thought experiments, Bekenstein boldly suggested that a black hole should possess an entropy proportional to its horizon area measured in Planck's units $G\hbar$ [5]. Hawking later showed by studying quantum fields in a black hole environment that a black hole horizon should radiate and should, therefore, possess temperature [6]. This temperature turned out to be

$$T = \frac{\hbar g_{BH}}{2\pi} = \frac{\hbar}{8\pi GM}. \qquad (1)$$

Here $g_{BH} = GMR_S^{-2}$ is the surface gravity at the horizon of a Schwarzschild black hole of mass M and radius R_S. Shortly after Hawking's discovery, it was realized that this result should not be restricted to black holes alone. Unruh showed that a uniformly accelerating observer through a Minkowski vacuum will perceive a heated horizon with temperature proportional to the observer's acceleration, $T = \hbar a / 2\pi$ [7]. These results further strengthened the thermodynamic description of horizons. With a combination of the area-mass relation, $A = 16\pi\left(GM\right)^2$, for a black hole and the first law of thermodynamics at the horizon, $T^{-1} = \Delta S / \Delta M$, the Hawking temperature formula leads to the black hole entropy as

$$S_{BH} = \frac{A}{4G\hbar}. \qquad (2)$$

This is called the Bekenstein-Hawking entropy formula. Equation (2), which is based on classical and semi-classical ideas culminates within itself gravity, quantum physics, and statistical mechanics and. 't Hooft [8] elevated this result to the status of a general principle – the so-called holographic principle – that is, quantum mechanics and general relativity requires that the three-dimensional information describing an isolated system in a region of space can be represented by the boundary of the region and is limited by the area of this boundary, with the number of microscopic degrees of freedom as finite and proportional to the area of the boundary in Planck's units.

One recalls that Padmanabhan [9,10] established an identity, $S = M / 2T$ relating the entropy of a general horizon to its temperature T and the active gravitational mass M the boundary encloses. Assuming that there are N Planckian degrees of freedom at the horizon, Padmanabhan's relation in combination with (2) can be reinterpreted as the equipartition of energy

$$M = \frac{1}{2} NT. \qquad (3)$$

Interestingly, Verlinde [11] conjectured that Newton's gravitational law can be shown to arise as an entropic force if one assumes that the thermodynamic degrees of freedom at a general holographic sphere obey the equipartition of energy as depicted in (3). Following Verlinde's holographic setup in the framework of loop quantum gravity (LQG) [12], Smolin came up with Newton's gravity by assuming the equipartition to hold (due mainly to spherical symmetry) at a general holographic sphere.

In this paper, we note that the value of the Immirzi parameter (γ) of LQG [13] used by Smolin in his derivation of Newton's gravity is inconsistent with the equipartition rule. In fact, as assumed by Smolin, if the equipartition rule holds for the LQG degrees of freedom at the black hole horizon or a general holographic sphere, then it leads to a different value of the γ parameter as compared to the values conceived previously [14,15]. In the following section, we first briefly comment on some of the earlier works regarding fixing the Immirzi parameter of LQG and then extend our new approach to ascertain the value of the γ parameter. Section 3 is devoted to discussion.

2. FIXING THE IMMIRZI PARAMETER γ OF LQG

Loop quantum gravity (LQG) is a canonical quantization of the classical gravitational field and uses spin networks as a basis for its Hilbert space [16,17]. Spin networks are graphs whose edges carry labels $\left\{ j \in 0, 1/2, 1, ... \right\}$ as the representations of the gauge group $SU(2)$ of the theory. Amongst various approaches to quantum gravity, LQG seems to be the sole theory that has produced results regarding geometrical spectra from the first principle. A key result of LQG is that the area of a given region of space is quantized in such a way that if a surface is punctured by an edge of the spin network, carrying a label j, the surface acquires an element of Planck sized area

$$A_j = \beta_j G\hbar. \quad (4)$$

Here $\beta_j = 8\pi\gamma\sqrt{j(j+1)}$ with γ as a free undetermined parameter of the theory, called the Immirzi parameter [13]. This parameter is known to have no effect on classical gravity but appears unavoidably in the quantized version of the theory. The actual physical meaning γ is still unknown. The theory however cannot produce useful predictions unless this parameter is worked out. The value γ is often fixed by comparing the LQG results with the known black hole dynamics. In [14,15], it was proposed that γ can be fixed by the requirement that the quantum gravity results reproduce the Bakenstein-Hawking entropy.

In LQG the degrees of freedom that determine the black hole entropy are the spin network edges puncturing the horizon. Statistically, the dominant contribution to the entropy comes from the lowest possible non-zero spin $j_{\min}$, so that the number of edges puncturing the horizon of the area A becomes

$$N_{j_{\min}} = \frac{A}{\beta_{j_{\min}} G\hbar}. \quad (5)$$

Given the multiplicity of the state $j_{\min}$ $(2j_{\min}+1)$ the entropy of the black hole is calculated as the logarithm of the dimension of the Hilbert space living on the horizon [14,15]:

$$S_{LQG} = \frac{A}{\beta_{j_{\min}} G\hbar}\ln(2j_{\min}+1). \quad (6)$$

A comparison of the LQG result (6) $j_{\min} = 1/2$ with the Bekenstein-Hawking formula (2) yields the value γ as

$$\gamma = \frac{\ln 2}{\pi\sqrt{3}}. \quad (7)$$

Dreyer [18] arrived at a different value of the Immirzi parameter by exploiting Hod's [19] semi-classical argument based on the quasinormal mode (QNM) spectrum of a Schwarzschild black hole [20,21]. Dreyer's argument led to the value of the Immirzi parameter as

$$\gamma = \frac{\ln 3}{2\pi\sqrt{2}}. \quad (8)$$

But this approach suggested that the dominant contribution should come from edges $j=1$ and that the true gauge group of the theory should be considered as *SO*(3) rather than *SU*(2). It remains obscure as to which group, *SU*(2) or *SO*(3), should now be adopted as the gauge group of the theory.

Following the LQG version of Verlinde's holographic set of equations, Smolin obtained the gravitational law in the form [12]

$$F = \frac{GM}{R^2}\left(\frac{\hbar}{\Delta x}\frac{2(\ln 2)^2}{\pi}\right). \quad (9)$$

Here M is the active gravitational mass enclosed by a holographic sphere. The quantity in the brackets including the dimensionless fudge factor $f = 2(\ln 2)^2/\pi$ was interpreted as representing the passive gravitational mass m of a particle near the holographic screen within its Compton wavelength approximately equal to Δx.

In the derivation of (9), the equipartition of energy

$$M = \frac{1}{2}N_{j_{\min}}T \quad (10)$$

was assumed due only to spherical symmetry. The value of the Immirzi parameter given by (7) was used in the derivation. But, as follows, one can readily prove that the use of the equipartition of energy at the black hole horizon or a general holographic sphere of radius R leads to a different value of the Immirzi parameter. Substituting $N_{j_{\min}}$ from (5), with $A = 4\pi R^2$, into (10) the temperature of the screen can be obtained as

$$T = \beta_{j_{\min}}\frac{\hbar g_R}{2\pi}. \quad (11)$$

This equation matches precisely with the Hawking-Unruh temperature formula provided one choses $\beta_{j_{\min}}$ as unity. Thus, γ must be chosen as

$$\gamma = \frac{1}{8\pi\sqrt{j_{\min}(j_{\min}+1)}} \quad (12)$$

if the equipartition of energy has to produce the correct temperature law. Assuming *SU*(2) to be the underlying group (that has to be fixed from elsewhere) and taking the $j_{\min} = 1/2$ edges as dominant, the value of γ can be fixed as

$$\gamma = \frac{1}{4\pi\sqrt{3}}. \quad (13)$$

With this new value of γ, Smolin's result (9) would be modified to the one with a new fudge factor $f = \ln 2/2\pi$.

3. DISCUSSION

The equipartition of energy imposed on the edges of LQG puncturing general horizon yields the correct Hawking-Unruh temperature formula provided the Immirzi parameter assumes the value given by (12), thereby yielding the actual operational area element to be exactly the Planck area. With this new value γ, one may suggest that the black hole entropy should be evaluated, not as equal, but proportional to the logarithmic measure of the dimensionality of the Hilbert space inducing the horizon metric. In this way, the LQG calculation for the black hole entropy wi $j_{\min} = 1/2$ ll effectively coincide with Bekenstein's formula [5].

We come across no direct comparison of a quantum result for entropy in (6) with the semi-classical result (2). Neither, at any point, does this formulation suggest an alteration of the theory's group structure. The assumption of the 'classical' equipartition in the quantum realm, however, is not that obvious. One may be tempted to delve deeply to explore how energy is distributed over the quantum degrees of freedom. The value of γ fixed at a general holographic screen should possess universal validity. However, it still remains obscured why is γ absent in classical gravity but appears unavoidably in the quantized version of the theory. It is also elusive as to what this parameter account for.

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